Spring Lecture Series N.48
Transport, Mixing and Fluids

Dates: May 5-7, 2023

Principal Lecturer: Anna Mazzucato (Penn State University)

Public Lecturer: Lisette de Pillis (Harvey Mudd College)

Invited Speakers:

Alexey Cheskidov (University of Illinois-Chicago)
Cecilia Mondaini (Drexel University)
Weiwei Hu (University of Georgia)
Siming He (University of South Carolina)
Andrej Zlatoš (University of California Santa Barbara)
Christian Seis (University of Munster)
Sam Punshon-Smith (Tulane University)
Theodore Drivas (Stony Brook University)
Zoran Grujić (University of Virginia)
Gautam Iyer (Carnegie-Mellon University)

Organizers: Ariel Barton (University of Arkansas), Zachary Bradshaw (University of Arkansas)

The University of Arkansas Spring Lecture series are conferences organized every spring by the Department of Mathematical Sciences of the University of Arkansas. Each conference is focused on a specific topic chosen among the current leading research areas in Mathematics; a principal lecturer delivers a short, five-lecture course and selects a number of specialists who are invited to give talks on subjects closely related to the topic of the conference. Short talks by young Ph.D.s and finishing graduate students are solicited to complement the conference. Each Lecture Series has grown into an ideal opportunity for specialists and young researchers to meet and exchange ideas about topics at the forefront of modern mathematics.

The Spring Lectures are usually sponsored by the NSF jointly with the University of Arkansas.

Abstracts and Talks:

Anna Mazzucato (Professor of Mathematics and Distinguished Senior Scholar of Penn State)
Title: [Principle Lecture] Numerical Linear Algebra: from Scientific Computing to Data Science Applications

Lisette de Pillis (Norman F. Sprague Professor of Life Sciences and Professor of Mathematics, Harvey Mudd College)
Title: [Public Lecture] Numerical Linear Algebra: from Scientific Computing to Data Science Applications
Abstract: *Mathematical models hold the keys to understanding some of the most interesting and complex phenomena in the natural world! In this talk, we will explore how to harness the power of mathematical modeling to answer challenging questions that may at first seem unsolvable. Can an overflowing bathtub help us figure out how to achieve herd immunity in a pandemic? Can the behavior of a lynx help us understand how human immune cells fight cancer? By making a few simplifying assumptions, we can draw parallels between natural systems that may appear radically different on the surface to unlock new levels of understanding the world around us.*

Weiwei Hu (University of Georgia, USA)
Title: Control Design for Optimal Mixing via Flow Advection
Abstract: *The question of what velocity fields effectively enhance or prevent transport and mixing, or steer a scalar field to the desired distribution, is of great interest and fundamental importance to the fluid mechanics community. In this talk, we mainly discuss the problem of optimal mixing of an inhomogeneous distribution of a scalar field via active control of the flow velocity, governed by the Stokes or the Navier-Stokes equations. Specifically, we consider that the velocity field is steered by a control input that acts tangentially on the boundary of the domain through the Navier slip boundary conditions. This is motivated by mixing within a cavity or vessel by rotating or moving walls. Our main objective is to design a Navier slip boundary control for achieving optimal mixing. Non-dissipative scalars governed by the transport equation will be our main focus. In the absence of molecular diffusion, mixing is purely determined by the flow advection. This essentially leads to a nonlinear control and optimization problem. A rigorous proof of the existence of an optimal control and the first-order necessary conditions for optimality will be addressed. Moreover, a feedback law (sub-optimal) will be also constructed based on interpolation of the optimality conditions. Finally, numerical experiments will be presented to demonstrate our ideas and control designs.*

Cecilia Mondaini (Drexel University, USA)
Title: Long-time statistics of SPDEs: mixing and numerical approximation
Abstract: *In analyzing complex systems modeled by stochastic partial differential equations (SPDEs), such as certain turbulent fluid flows, an important question concerns their long-time behavior. In particular, one is typically interested in determining how long it takes for the system to settle into statistical equilibrium, and in investigating efficient numerical schemes for approximating such long-time statistics. In this talk, I will present two general results in this direction, and illustrate them with an application to the 2D stochastic Navier-Stokes equations. Most importantly, our approach does not require gradient bounds for the underlying Markov semigroup as in previous works, and thus provides a flexible formulation for further applications.*

This is based on joint work with Nathan Glatt-Holtz (Tulane U).

Zoran Grujić (University of Virginia, USA)
Title: On criticality of the Navier-Stokes diffusion
Abstract: *The main purpose of this talk is to present a mathematical evidence of criticality of the Navier-Stokes diffusion. In particular, considering a plausible candidate for a finite time blow-up, a two-parameter family of the dynamically rescaled profiles, we show that as soon as the hyper-diffusion exponent is greater than one, a new region in the parameter space (completely in the super-critical regime) is ruled out. As a matter of fact, the region is a neighborhood (in the parameter space) of the self-similar profile, i.e., the 'approximately self-similar' blow-up is ruled out for all hyper-diffusive models.*

Alexey Cheskidov (University of Illinois-Chicago, USA)
Title: Turbulent solutions of fluid equations
Abstract: *Computing the determinant of a large-scale symmetric positive definite matrix A is a task that arises in many applications, for example in the training of a Gaussian process regression model. When the matrix is sufficiently small, its determinant can be computed via a Cholesky decomposition of A. When A is large and this approach is too costly, one can still get an approximation of the determinant by estimating the trace of a suitable matrix, that is, the matrix logarithm log(A), using randomized algorithms. In this lecture we consider the Hutchinson trace estimator, which obtains an approximation to the trace of a matrix B by averaging some quadratic forms involving B and random vectors following a suitable distribution. In the context of determinants, the advantage of this approach is that quadratic forms involving log(A) can be approximated efficiently using Lanczos method. We discuss convergence bounds for the Hutchinson trace estimator, focusing on the case in which B is a symmetric but indefinite matrix, and apply the bounds to the approximation of the determinant.*

Christian Seis (University of Münster, Germany)
Title: Mixing by Randomly Driven Vortices
Abstract: *We consider passive scalar transport in a two-dimensional domain in the case where the velocity is generated by a randomly moving vortex. Using purely Eulerian arguments, we prove that the velocity field is exponentially mixing. Moreover, in the presence of diffusion, we show enhanced dissipation at a rate that is independent of the diffusivity constant.*
This is joint work with Víctor Navarro-Fernández and André Schlichting.

Siming He (University of South Carolina, USA)
Title: Enhanced dissipation and blow-up suppression in a chemotaxis-fluid system
Abstract: *In this talk, we will present a coupled Patlak-Keller-Segel-Navier-Stokes (PKS-NS) system that models chemotaxis phenomena in the fluid. The system exhibits critical threshold phenomena. For example, if the total population of the cell density is less than 8π, then the solutions exist globally in time. Moreover, finite time blowup solutions exist if this population constraint is violated. We further show that globally regular solutions with arbitrary large cell populations exist. The primary blowup suppression mechanism is the shear flow mixing induced enhanced dissipation phenomena.*

Theodore Drivas (Stony Brook University, USA)
Title: Irreversible Features of the 2D Euler Equations
Abstract: *We will discuss aspects of the long term dynamics of 2d perfect fluids. As an application of a certain stability of twisting for general hamiltonian flows, we will show generic loss of smoothness near stable steady states, the existence of many wandering points, aging of the Lagrangian flow, along with other examples of complex behavior such as indefinite perimeter growth for special vortex patches.*

Sam Punshon-Smith (Tulane University, USA)
Title: Advection Diffusion by Markovian Velocity Fields: Chaos, Mixing and Norm Equivalence of Decay Rates
Abstract: *In this talk I will survey several results on the mixing and decay properties of passive scalars advected by ergodic Markovian velocity fields including, but not limited to, the stochastic Navier-Stokes equations in 2D. As a general principle, when such a velocity field is sufficiently regular and non-degenerate (in the sense that it explores a large enough set of velocity fields) then one should expect the corresponding Lagrangian flow to be both chaotic (has a positive Lyapunov exponent) and exponentially mixing. In addition, the mixing rate will be stable under the addition of diffusion, despite being a singular perturbation of the associated scalar equation. I will explain a general framework using rigidity of group actions and spectral properties of certain Markov semi-groups where this principle can be rigorously verified in the case of stochastically forced fluids as well as in the case of random alternating shears (Pierrehumbert flow). I will then present a general condition under which the asymptotic decay rates for a compact linear evolution are norm independent. Applying this to the advection diffusion equation implies that the asymptotic decay rates are independent of the norm used to measure decay (i.e. H2, L2, H-1). Therefore upper and lower bounds on the decay rates in one norm (e.g. mixing) immediately implies bounds on the decay rates in any other norm.*
This work is joint with Jacob Bedrossian and Alex Blumenthal.

Andrej Zlatoš (University of California San Diego, USA)
Title: Euler equations on general planar domains
Abstract: *Bounded vorticity solutions to the 2D Euler equations on singular domains are typically not close to Lipschitz near boundary singularities, which makes their uniqueness a difficult open problem. I will present a general sufficient condition on the geometry of the domain that guarantees global uniqueness for all solutions initially constant near the boundary. This condition is only slightly more restrictive than exclusion of corners with angles greater than π and, in particular, is satisfied by all convex domains. Its proof is based on showing that fluid particle trajectories for general bounded vorticity solutions cannot reach the boundary in finite time. The condition also turns out to be sharp in the latter sense: there are domains that come arbitrarily close to satisfying it and on which particle trajectories can reach the boundary in finite time. The above results also extend to signed vorticity solutions on fairly irregular domains that may even contain infinitely many corners with angles greater than π.*

Gautam Iyer (Carnegie-Mellon University, USA)
Title: Enhanced Dissipation and Mixing
Abstract: *In many systems where convection and diffusion are both present, they work together and enhance energy dissipation. I will talk about how enhanced dissipation can be used to control certain nonlinear effects, and how enhanced dissipation can be quantified using both deterministic and probabilistic techniques.*

UNIVERSITY OF ARKANSAS

Department of Mathematics 48th Annual Spring Lecture Series on

Transport, mixing and fluids

May 5-7, 2023

A Series of 5 Lectures by

Anna Mazzucato (Penn State University)

With supporting lectures by:

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Gautam Iyer (Carnegie-Melon University)

Featuring a public lecture by:

Lisette de Pillis (Harvey Mudd College)

Partial funding is available for junior participants.
Find registration, schedule and more information here:
https://aeb019.hosted.uark.edu/spring-lecture-series-2023.html
Contacts: Ariel Barton (aeb019@uark.edu) or Zachary Bradshaw (zb002@uark.edu).