49th Annual Spring Lecture Series

Harmonic analysis, partial differential equations, and geometric measure theory

May 3–5, 2024

The conference will take place in:

- SCEN (Science Engineering Hall) 407 on the morning of May 3rd (Map)
- Hillside 206 on the afternoon of May 3rd, all day May 4 and the morning of May 5. (Map). This includes the women in math panel and the public lecture.

Please register here! Partial travel funding for selected participants will be supported by a grant from the NSF.

This conference is in Cooperation with AWM and supports the Welcoming Environment Statement of the Association for Women in Mathematics.

Principal Speaker: **Svitlana Mayboroda**
McKnight Presidential Professor, University of Minnesota

Public Lecture | Change of Perspective in Mathematics

May 3, 2024 (5:30 pm CDT) in Hillside 206

Public Lecturer: **Michael Orrison**
Professor of Mathematics, Harvey Mudd College

Note: No registration is necessary for the public lecture.

Women in Mathematics Panel

Friday, May 3, 2024 (5:00 pm CDT) in Hillside 206

Invited Speakers

Matthew Badger (University of Connecticut)

Blair Davey (Montana State University)

Guy David (Université de Paris-Sud (Orsay))

Joseph Feneuil (Australian National University)

Silvia Ghinassi (University of Washington)

Steve Hofmann (University of Missouri)

José María Martell (Instituto de Ciencias Matemáticas (ICMAT))

Zihui Zhao (Johns Hopkins University)

Organizers

Ariel Barton (aeb019@uark.edu)
Associate Professor of Mathematical Sciences, University of Arkansas

Zachary Bradshaw (zb002@uark.edu)
Associate Professor of Mathematical Sciences, University of Arkansas

Schedule of Talks

	Friday, May 3th	**Saturday, May 4th**	**Sunday, May 5th**
8:30	Registration—Coffee/Tea SCEN 407	Coffee/Tea Hillside 206	Coffee/Tea Hillside 206
8:55	Opening remarks		
9:00	Guy David Université de Paris-Sud (Orsay) Zoom	Silvia Ghinassi University of Washington Hillside 206	Short talks by early career Mathematicians Hillside 206
10:00	Svitlana Mayboroda University of Minnesota and ETH Zurich (Lecture #1) SCEN 407	Svitlana Mayboroda University of Minnesota and ETH Zurich (Lecture #3) Hillside 206	Svitlana Mayboroda University of Minnesota and ETH Zurich (Lecture #5) Hillside 206
11:00	Blair Davey Montana State University SCEN 407	Steve Hofmann University of Missouri Hillside 206	Joseph Feneuil Australian National University Hillside 206
12:00	Lunch	Lunch	Shuttle to Airport
2:30	Svitlana Mayboroda University of Minnesota and ETH Zurich (Lecture #2) Hillside 206	José María Martell Instituto de Ciencias Matemáticas (ICMAT) Zoom	
3:30	Matthew Badger University of Connecticut Hillside 206	Svitlana Mayboroda University of Minnesota and ETH Zurich (Lecture #4) Hillside 206	
4:30		Zihui Zhao	

5:00	Women in Mathematics panel Hillside 206	Johns Hopkins University Hillside 206
5:30	Public Lecture Michael Orrison, Harvey Mudd College Hillside 206	Walk to dinner

Abstracts of Talks

Matthew Badger, University of Connecticut

Title: Nodal Domains of Homogeneous Caloric Polynomials

Abstract: With a view towards confirming the existence of singular strata in Mourgoglou and Puliatti's two-phase free boundary regularity theorem for caloric measure, we identify the minimum number of nodal domains of homogeneous caloric polynomials (hcps) in $\mathbf{R}^{n+1}$ of degree d. We also provide estimates on the maximum number of nodal domains for all n and d. I'll survey the techniques that go into the proofs of the theorems, particularly the construction of hcps that realize the minimum number of nodal domains. This is joint work with Cole Jeznach.

Blair Davey, Montana State University

Title: Fractional parabolic theory as a high-dimensional limit of fractional elliptic theory

Abstract: Experts have long realized the parallels between elliptic and parabolic theory of partial differential equations. It is well-known that elliptic theory may be considered a static, or steady-state, version of parabolic theory. And in particular, if a parabolic estimate holds, then by eliminating the time parameter, one immediately arrives at the underlying elliptic statement. Producing a parabolic statement from an elliptic statement is not as straightforward. In this talk, we discuss how a high-dimensional limiting technique can be used to prove theorems about solutions to the fractional heat equation (or its Caffarelli-Silvestre extension problem) from their elliptic analogues. This talk covers joint work with Mariana Smit Vega Garcia.

Guy David, Université de Paris-Sud (Orsay)

Title: Counterexamples involving elliptic measure and Cantor sets

Abstract: We'll describe examples of Cantor sets or snowflakes and elliptic operators for which the elliptic measure on the given set is equivalent to the natural Hausdorff measure. This is joint work with Jeznach, Julia, Mayboroda, and Perstneva.

Joseph Feneuil, Australian National University

Title: Green functions, smooth distances, and uniform rectifiability

Abstract: The past 10 years have seen considerable achievements at the intersection of harmonic analysis, PDE, and geometric measure theory. One now better understands the

relationship between the geometry of the boundary of a domain and the regularity of harmonic/elliptic solutions inside the domain. For instance, it was proved that the uniform rectifiability (UR) of a codimension 1 set is characterized by the A_∞-absolute continuity of its harmonic measure with respect to the surface measure - or equivalently the solvability of a L^p Dirichlet problem in the complement.

In this talk, I will show that another characterization of UR sets of codimension 1 can be obtained by comparing the Green function G with some regularized version of the distance to the boundary. Moreover, I will obtain a characterization of any UR set of any codimension by an estimate on $\nabla|\nabla G|$. Those are joint works with Guy David, Linhan Li, and Svitlana Mayboroda.

Silvia Ghinassi, University of Washington

Title: Self-similar sets and Lipschitz graphs

Abstract: We investigate and quantify the distinction between rectifiable and purely unrectifiable 1-sets in the plane. That is, given that purely unrectifiable 1-sets always intersect Lipschitz objects on a set of 1-measure zero, we ask whether these sets overlap with Lipschitz images or graphs at a dimension that is close to one. In an answer to this question, we show that one-dimensional attractors of iterated function systems satisfying the open set condition have subsets of dimension arbitrarily close to one that can be covered by Lipschitz graphs. Moreover, the Lipschitz constant of such graphs depends explicitly on the difference of the dimension of the original set and the subset that overlaps with the graph. This is joint work with Blair Davey and Bobby Wilson.

Steve Hofmann, University of Missouri

Title: A problem of free boundary type for caloric measure

Abstract: For an open set $\Omega\subset\mathbf{R}^d$ with an Ahlfors regular boundary, solvability of the Dirichlet problem for Laplace's equation, with boundary data in L^p for some $p<\infty$, is equivalent to quantitative, scale invariant absolute continuity (more precisely, the weak-A_∞ property) of harmonic measure with respect to surface measure on $\partial\Omega$. A similar statement is true in the caloric setting. Thus, it is of interest to find geometric criteria which characterize the open sets for which such absolute continuity (hence also solvability) holds. Recently, this has been done in the harmonic case. In this talk, we shall discuss recent progress in the caloric setting, in which we show that quantitative absolute continuity of caloric measure, with respect to "surface measure" on the parabolic Ahlfors regular (lateral) boundary Σ, implies parabolic uniform rectifiability of Σ. We observe that this result may be viewed as the solution of a certain 1-phase free boundary problem.

This is joint work with S. Bortz, J. M. Martell and K. Nyström.

José María Martell, Instituto de Ciencias Matemáticas (ICMAT)

Title: The Dirichlet problem with data in Hölder spaces in rough domains

Abstract: We consider the Dirichlet problem for real-valued second order divergence form elliptic operators with boundary data in Hölder spaces. We work in open sets satisfying the capacity density condition (a quantitative version of the Wiener), without any further topological assumptions such as connectivity, and show that the Dirichlet boundary value problem is well-posed for boundary data in Hölder spaces with small enough exponent if Ω is

either bounded, or unbounded with unbounded boundary. However, when Ω is unbounded with bounded boundary (e.g., the complement of a compact set), we establish that solutions exist, but they fail to be unique in general. These results are optimal in the sense that solvability of the Dirichlet problem in Hölder spaces is shown to imply the capacity density condition.

Zihui Zhao, Johns Hopkins University

Title: Boundary unique continuation of harmonic functions

Abstract: Unique continuation property is a fundamental property for harmonic functions, as well as a large class of elliptic and parabolic PDEs. It says that if a harmonic function vanishes at a point to infinite order, it must vanish everywhere. In the same spirit, we are interested in quantitative unique continuation problems, where we use the local growth rate of a harmonic function to deduce some global estimates, such as estimating the size of its singular or critical set. In this talk, I will talk about some recent results together with C. Kenig on boundary unique continuation.

Updated 5/2/2024